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AN UNCERTAINTY ANALYSIS OF PHOTOMETRIC RESPONSIVITY BASED ON SPECTRAL IRRADIANCE RESPONSIVITY

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Abstract

The uncertainty for photometric responsivity calibrated with the tunable lasers in photometry setup (TULIP) at Physikalisch-Technische Bundesanstalt (PTB) is presented. The measurement and the uncertainty calculation presented were done in preparation for the upcoming new traceability chain for luminous intensity at PTB. Regarding the new traceability a comprehensive uncertainty calculation is needed to take into account all steps of the traceability chain. The components of the measurement model are described and the correction factor for bandwidth and the inclusion of correlations are evaluated in detail. Assumptions that can help accessing unknown spectral correlations are described and their effect on the measurement uncertainty of photometric responsivity is calculated.

Keywords: Photometry, Spectroradiometry, Measurement Uncertainty, Bandwidth, Correlations

1 Motivation

For a more direct radiometric traceability of luminous intensity a new detector was developed at PTB during the last years. The configuration and components of this so-called $V(\lambda)$ -trap detector were chosen carefully regarding the demands of each step of the traceability chain and have been previously described (Schneider 2018). For testing the detector's capabilities, a calibration of its spectral responsivity and luminous responsivity were done at the TULIP setup of PTB (Schuster 2014) using the current traceability chain. The key part of this calibration was creating a measurement model and the corresponding uncertainty evaluation.

Aim of the measurement model was to establish a calculation not specific to the old traceability chain but versatile enough to be adopted to the upcoming new traceability chain at PTB using only the $V(\lambda)$ -trap detector. The required calculation of the measurement uncertainties of integral quantities, the photometric responsivity in this case, has been part of extensive research already (Wooliams 2013, Winter 2006) including correlations (Gardner 2004). Based on the literature and the demanded versatility a Monte Carlo approach was chosen for evaluating the measurement results and for the calculation of the associated measurement uncertainty.

2 Measurement model

The measurement model is based on a substitution measurement comparing the device-undertest (DUT) detector with the calibrated reference (REF) detector. The model and the resulting uncertainty budget is not complete, as for example the angular responsivity change of the detector, temperature dependency or linearity are not included yet. Still, the major components have been included in addition to some smaller contributions that have been measured while developing the detector.

The model is as follows:

$$s_E(\lambda) = \frac{U_{\text{DUT}} \cdot U_{\text{Mon,REF}}}{U_{\text{REF}} \cdot U_{\text{Mon,DUT}}} \cdot \frac{R_{\text{REF}}}{R_{\text{DUT}}} \cdot s_{\phi,\text{REF}} \cdot (\lambda) \cdot A_{\text{REF}} \cdot c_{\text{wl}}(\lambda) \cdot c_{\text{bw}}(\lambda) \cdot c_{\text{pol}}(\lambda) \cdot c_{\text{unif}}(\lambda) \cdot c_{\text{dist}}$$
(1)

where

 $s_E(\lambda)$ is the spectral irradiance responsivity of the DUT

$U_{\rm DUT/REF}$	are the voltage readings when measuring either the DUT- or REF-detector
U _{Mon,x}	are the simultaneous voltage readings of the monitor detector with respect to the DUT- or REF signal
$R_{\rm REF/DUT}$	are the calibrated resistances of the used photocurrent amplifier
$s_{\Phi,\mathrm{REF}}(\lambda)$	is the spectral power responsivity of the reference detector
$A_{\rm REF}$	is the area of the aperture in front of the reference detector
$c_{\rm wl}(\lambda)$	is the correction factor for the wavelength measurement
$c_{\rm bw}(\lambda)$	is the correction factor for bandwidth effects
$c_{\mathrm{pol}}(\lambda)$	is the correction factor for polarization dependency of the DUT
$c_{\mathrm{unif}}(\lambda)$	is the correction factor for non uniformity of DUT, REF and the used radiance
<i>c</i> _{dist}	is the correction factor for distance offsets between DUT and REF

The uniformity of the $V(\lambda)$ -trap detector (Schneider 2015) and its polarization dependency (Schneider 2018) have been described before. Calculation of the resulting correction factors will be described in the following. Starting with the correction factor of the wavelength which is set to unity, as the wavelength is directly measured with a wavelength calibrated spectroradiometer for each measurement. To take into account the uncertainty of the wavelength measurement λ_{spec} following formalism is used:

$$u(c_{\rm wl}(\lambda)) = 1 - \frac{1 - u(\lambda_{\rm spec}) \cdot s'_{\rm DUT}(\lambda) / s_{\rm DUT}(\lambda)}{1 - u(\lambda_{\rm spec}) \cdot s'_{\rm REF}(\lambda) / s_{\rm REF}(\lambda)}$$
(2)

where

$$\begin{array}{ll} u(c_{\rm wl}(\lambda)) & \text{is the uncertainty contribution of the wavelength uncertainty} \\ u(\lambda_{\rm spec}) & \text{is the uncertainty of the measured wavelength} \\ s'_{\rm DUT/REF}(\lambda) & \text{are the first derivatives of the spectral responsivities of DUT and REF detector} \\ s_{\rm DUT/REF}(\lambda) & \text{are the spectral responsivities of DUT and REF detector} \end{array}$$

The maximum change in the detectors responsivity due to polarization Δs_{pol} presented in (Schneider 2018) can be directly used for the correction factor:

$$c_{\rm pol}(\lambda) = 1 + \Delta s_{\rm pol} \cdot P_{\rm pol}(\lambda) \cdot \sin(2\pi \cdot \varphi) \tag{3}$$

where

 $P_{\rm pol}(\lambda)$ is the degree of linear polarization of the incident radiant flux

 φ is the polarization angle

The uncertainty of $c_{pol}(\lambda)$ is calculated straightforward by propagating the uncertainties of the contributing parameters.

The correction factor for nonuniformities of the detectors and the incident radiation is calculated using the surface integral:

$$c_{\text{unif}}(\lambda) = \left[\oint_{A} s_{\text{rel,DUT}}(x, y, \lambda) dx dy / \oint_{A} s_{\text{rel,DUT}}(x, y, \lambda) \cdot E_{\text{rel}}(x, y, \lambda) dx dy \right] \cdot \left[\oint_{A} s_{\text{rel,ref}}(x, y, \lambda) \cdot E_{\text{rel}}(x, y, \lambda) dx dy / \oint_{A} s_{\text{rel,ref}}(x, y, \lambda) dx dy \right]$$
(4)

where

$s_{\rm rel,DUT/REF}(x, y, \lambda)$	are the locally resolved relative spectral responsivity distributions of DUT ar	۱d
	REF detector	

$$E_{rel}(x, y, \lambda)$$
 is the locally resolved relative spectral irradiance distribution in the measurement plane

The uncertainty of $c_{unif}(\lambda)$ is calculated to propagate the uncertainties of the irradiance distributions and responsivity distributions.

The correction factor for distance offsets is set to unity because the detectors are measured in the same nominal distance to the source of the radiation field. However, the deviation from ideal alignment contributes to the uncertainty:

$$u(c_{\rm dist}) = \sqrt{\left(\frac{u(d_{\rm DUT})}{d_0}\right)^2 + \left(\frac{u(d_{\rm REF})}{d_0}\right)^2} \tag{5}$$

where

 $d_{\text{DUT/REF}}$ are the distances of the DUT and ref from the source

 d_0 is the nominal distance between detector and source

More detailed considerations about the correction factors and the uncertainty calculation are given in (Schneider 2 2018).

The correction factor for bandwidth effects is described separately in section 2.1 as there are two separate approaches presented and compared. The possibility to include covariance data of the reference detector has been included in the calculation when determining the reference detectors responsivity values. The used formalism is described in section 2.2.

2.1 Bandwidth correction

The bandwidth of the radiation used to measure the responsivity of any detector becomes important when there is a change in the gradient of the responsivity within the range of the bandpass. Thus, for a linear responsivity of the detector or for a small bandwidth of the source, i.e. cw lasers, bandwidth correction may become neglectable. For the calibration of the $V(\lambda)$ -trap detector a filtered detector was investigated. For these measurements the bandwidth-limited radiation of a pulsed femtosecond-laser setup with $\Delta \lambda = 1 \text{ nm}$, provided by a monochromator, was used. The bandwidth was measured with an echelle grating spectrometer to be within 0,9 nm to 1,1 nm for the whole spectral range and to be of a symmetric, approximately triangular profile. These conditions require the correction of bandwidth effects.

One possibility of calculating the bandwidth correction already used at PTB for calibration of photometers (Winter 2006) and also mentioned in CIE (CIE 2014) is using the second derivative of the detectors responsivity:

$$c_{\rm bw}(\lambda) = \frac{\frac{1-\Delta\lambda^2 \cdot \frac{1}{12} s_{\rm REF}^{\prime\prime}(\lambda)}{s_{\rm REF}(\lambda)}}{\frac{1-\Delta\lambda^2 \cdot \frac{1}{12} s_{\rm DUT}^{\prime\prime}(\lambda)}{s_{\rm DUT}(\lambda)}}$$

(6)

where

 $\Delta \lambda$ is the FWHM bandwidth of the radiation

 $s''_{x}(\lambda)$ is the second derivative of the DUT or REF detector responsivity

With this formalism the influence of a triangular bandpass on a known detector responsivity can be calculated and compared to the effect of the bandpass on another known responsivity. Therefore, a prior knowledge of the DUT and REF responsivities is required. They must first be determined without correction. As stated in (CIE 2014) the formalism can be used for a triangular bandpass, which is fulfilled for the measurement at hand. Regarding the uncertainty calculation the correction can be calculated for uniform distribution of the bandpass width between the given values within the Monte Carlo approach. For the $V(\lambda)$ -trap detector, calibrated against a typical three-element reflection trap detector built from Hamamtsu S1337 photodiodes, the correction factor is plotted in Figure 1.



Figure 1 – Bandwidth correction factor calculated with the derivative approach

A different approach to calculating a bandpass correction is described by Eichstädt et al. (Eichstädt 2013). The Richardson-Lucy deconvolution method originally described by Eichstädt et al. for spectrometer bandpass correction, can be applied. Measuring the responsivity of the detector with a limited bandwidth radiation is a similar convolution than in a spectrometer. In the approach, the original responsivity of the measured detector is iteratively calculated by repeated convolution of the responsivity spectra with the bandpass function. The bandpass determined by measurement can be used directly or it can be mathematically modelled as irregular, triangular, etc. Due to the deconvolution, the responsivity data needs to be padded at the ends of the spectral range. The padding values must be chosen carefully to avoid generation of fast oscillating values caused by discontinuities of the input values. In Figure 2 the correction factor calculated with this approach is shown. At 360 nm a significant edge is visible, resulting from the padding values.



Figure 2 – Bandwidth correction factor calculated with the Richardson-Lucy approach

For the Richardson-Lucy method the uncertainty can also be calculated as described for the derivation approach by alternating the bandpass function used for calculation within the Monte Carlo approach. The iterative convolution requires several iterations to approach a chosen level of deviation between the result of the calculation and the estimated responsivity values.

Therefore, it requires extensive iterative calculation if bandpass functions are changing within the wavelength range.

Nonetheless both approaches for calculation provide very similar results and can indubitably be exchanged for this case at hand. The strong changes in the correction factor at 800 nm are caused by noise in the signal and are well below the measurement uncertainty of about 10% (see Figure 7, Section 3). Due to the integral calculation of photometric responsivity the large correction factor and uncertainty do not contribute majorly to the uncertainty of photometric responsivity. For other detectors with different responsivity functions, it must again be checked whether the simplifications of the derivation approach can still be used.

2.2 Covariance matrices for spectral responsivity

Measurement values of spectral distributions, such as responsivity values of detectors or spectra determined with a spectroradiometer, are typically correlated. Correlation can be introduced, for example, by a common offset of the wavelength scale of the used monochromators or by the usage of mathematical models for interpolation of the values.

When calculating integral quantities, e.g. the photometric responsivity of a detector, from spectral irradiance responsivity data, correlations directly influence the measurement uncertainty. Depending on the correlation the integral measurement uncertainty can be either reduced or increased.

To access the correlated values for the Monte Carlo simulation, the values must be drawn according to the distributions and correlations. If the correlations are known, accessible or if they can be estimated, a multivariate normal copula function can be used (Possolo 2010). This way, the univariate distributions of all correlated values can be combined to a cumulative distribution using the correlation matrix. Given spectral responsivity values of a detector, each with a corresponding standard uncertainty can be combined with the correlation matrix to access correlated values for uncertainty calculation by a Monte Carlo method.

For the current evaluation, the correlations are estimated based on the principal correlations shown by Winter and Sperling for a monochromator-based measurement (Winter 2006), as the reference detectors used at the TULIP setup were calibrated using a comparable monochromator-based measurement setup. Also, a similar change of the bandwidth indicating a change of the grating of the monochromator at 700 nm occurs for the reference detectors at hand.

For estimating the correlation matrix the description of the measurement in the calibration certificate and the uncertainty of the responsivity can be used. For the reference detectors at hand the change of bandwidth in the calibration is described at 700 nm. The comparison to the similar measurements in (Winter 2006) with known correlation lead to the conclusion to assume full correlation both above and below 700 nm due to wavelength offset of the grating of the monochromator. The correlated, non-correlated (Figure 3) or negative correlated. All cases, together with no correlations (Figure 4) at all, are compared for the detector at hand, looking at the photometric responsivity and its uncertainty in Table 1.

A more detailed approach to the correlations was used for Figure 5 and for the final determination of the correlation. The described measurement procedure with a change in bandwidth results in the already shown change of correlation at around 700 nm. The uncertainty of the calibration values also shows a significant increase below 380 nm (Figure 7). This was due to a defect of one of the reference detectors during the measurement. Therefore, only a single reference detector was used below 380 nm, where as two reference detectors were used in the range 380 nm to 700 nm. Hence, a positive correlation between the values below 380 nm and above 380 nm can be assumed, but not as strong as with both reference detectors. These assumption lead to the correlation matrix shown in figure 5.

The result for the photometric responsivity is $9,358 \cdot 10^{-9} \frac{A}{lx}$ for all cases described above with the relative standard uncertainties given in Table 1. The uncertainty and correlation for the detailed approach were also used in the final calculation of the uncertainty and is described in section 3.

It is apparent that including correlations over parts of the spectrum slightly increases the measurement uncertainty in the case with detailed correlations used for the final correlation and cannot be easily neglected for integral quantities. The goal of the new traceability chain for luminous intensity at PTB is to achieve a minimal measurement uncertainty. When comparing the contributions to the uncertainty the radiometric calibration of the photometer was dominating compared to the lamp contribution. With a measurement uncertainty for photometric responsivity in the $5 \cdot 10^{-4}$ range, both contributions are in the same range. Therefore the uncertainty increase due to correlation cannot be neglected here. In the case at hand the uncertainty for photometric responsivity increases from $5,14 \cdot 10^{-4}$ to about $6,3 \cdot 10^{-4}$ for every correlation model. This is mainly due to the positive correlation of the spectral range below 700 nm which is photometrically more relevant than the upper spectral range.

Correlation	Relative standard uncertainty	
No correlation (figure 4)	$5,14 \cdot 10^{-4}$	
Full correlation	$6,29 \cdot 10^{-4}$	
Partially full correlation (figure 3)	$6,33 \cdot 10^{-4}$	
Partially full and negative (0,5) correlation (like figure 3 but -0,5 correlation)	6,29 · 10 ⁻⁴	
Full positive and full negative correlation (like figure 3 but -1 correlation)	6,31 · 10 ⁻⁴	
Detailed correlation assumption (figure 5)	$6,12 \cdot 10^{-4}$	

able 1 – Uncertainty components	s for luminous	responsivity of	the $V(\lambda)$ -trap detector
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Figure 3 –Correlation matrix with full correlation for values above and below 700 nm and no correlation between the two spectral ranges



Figure 4 – Correlation matrix for no correlations but the measurement points of the reference detector to themselves



Figure 5 – Detailed correlation matrix based on the calibration description and the measurement uncertainty



3 Results and measurement uncertainty budget

Figure 6 – spectral irradiance responsivity of the $V(\lambda)$ -trap detector with relative standard uncertainty

The spectral irradiance responsivity of the $V(\lambda)$ -trap detector and the total relative uncertainty following Equation 1 are plotted in Figure 6. A significant increase in uncertainty can be seen towards the UV and NIR spectral ranges. This coincides with the low responsivity spectral ranges of the detector, though further conclusions cannot be done from this data. Accessing the single components of a measurement uncertainty budget can be done by including additional steps in the Monte Carlo calculation. Within the calculation all components of the measurement model affecting the uncertainty are changed. Fixing all but one component to their mean values during Monte Carlo simulation gives information about the sensitivity coeffection of the very components, as the resulting distribution represents the effect of only the single component on the total distribution.



Figure 7 – spectral uncertainty plot for the irradiance responsivity of a photometric detector

The data can be plotted spectrally resolved as shown in Figure 7 to get an overview of which components in which wavelength range dominate the uncertainty. From this data it can be concluded that the signal-to-noise-ratio of the measurement itself is dominating by far in the near IR and UV range, depicted as U_{mess} in Figure 7, where U_{mess} summarizes the voltage values

of equation 1. When integrating the irradiance responsivity to calculate the luminous responsivity the signal-to-noise-ratio of the irradiance measurement remains one of the larger contributions as can be seen in Table 2.

Table 2 – Relative standard uncertainty components for luminous responsivity of th	е
$V(\lambda)$ -trap detector	

Uncertainty component	Relative standard uncertainty
Aperture area	$4,2 \cdot 10^{-4}$
Reference detector responsivity	$3,6 \cdot 10^{-4}$
Irradiance measurement	2,8 · 10 ⁻⁴
Wavelength measurement	$1,7 \cdot 10^{-5}$
Non-Uniformity	$1,4 \cdot 10^{-5}$
Further components	$1,0 \cdot 10^{-5}$
Combined uncertainty	6, 1 · 10 ⁻⁴

When comparing the bandwidth correction factor to the total uncertainty of the responsivity of the detector given in figure 3, the major deviations from unity correction factor are in spectral ranges with high associated uncertainty. The uncertainty of the measured signals was not taken into account in the calculation of the correction factor and its uncertainty, so the bandwidth uncertainty is probably higher. But as the major changes in the detector's responsivity are between 450 nm and 650 nm, a higher bandwidth uncertainty would not significantly add to the uncertainty of luminous responsivity

4 Conclusion

The Monte Carlo based calculation of the measurement uncertainty proved to be a versatile tool that can be easily expanded to include additional correction factors and steps. It is possible to access the contributions of the single quantities to the total measurement uncertainty. This way, identifying limiting components is possible. For the measurement at hand the limits are given by the knowledge of the aperture area, the reference detectors responsivity and the limits of the setup itself in terms of signal-to-noise-ratio.

The contribution of the aperture area and of the reference detector will be reduced in future by a new traceability chain that is currently established at PTB and that will be reported on soon. The uncertainty calculation presented will be modified to include all additional steps from power responsivity to luminous intensity measurement with the $V(\lambda)$ -trap detector. A similar expansion of the model has already been done for calculating LED illuminances with a detector calibrated with the TULIP setup of PTB following the measurement model of equation 1.

Two approaches for the correction of bandwidth effects have been applied and evaluated. The Richardson-Lucy approach can be applied without assumption about the bandpass function. The calculation needs more extensive calculation and is prone to padding values needed for the convolution. The derivative approach using the second derivative of the detector's responsivity provides a more stable calculation. The second method presumes a symmetric and triangular shaped bandpass. The Richardson-Lucy method can therefore be used to show the applicability of the assumptions used for the measurement at hand.

The influence on the resulting measurement uncertainty was shown, when calculation integral quantities from spectral measurement where spectral correlations are included. For the case at

hand the assumption of full correlation for the reference responsivity already yields an increase in measurement uncertainty comparable to more detailed correlation matrices.

References

- SCHNEIDER, P. et al 2018. Improved calibration strategy for luminous intensity. *Journal of Physics: Conf. Series.*, 972
- SCHUSTER, M. et al 2014. Validation of short-pulse-laser-based measurement setup for absolute spectral irradiance responsivity calibration. *Applied Optics*. 53, 13, 2815-2821
- WOOLIAMS, E. 2013. Determining the uncertainty associated with integrals of spectral quantities. *Report in EMRP-ENG05-1.3.1*
- GARDNER, J. 2004. Uncertainty Propagation for NIST Visible Spectral Standards. J. Res. Natl. Inst. Stand. Technol., 109, 305-318
- CIE 2014. CIE214:2014. Effect of Instrumental Bandpass Function and Measurement Interval on Spectral Quantities. Vienna: CIE
- SCHNEIDER, P. et al 2015. Developing a new detector for the realization of the SI base unit "candela". *CIE* x041:2016, 62-70
- SCHNEIDER, P. 2018. Aufbau und Charakterisierung eines neuartigen photometrischen Trapempfängers zur Realisierung der Basiseinheit der Lichtstärke Candela. *PTB-Bericht Opt 82*, Braunschweig: Fachverlag NW
- WINTER, S. et al 2016. Uncertainty Analysis of a photometer calibration at the DSR setup of the PTB. *CIE x029:2006*, 139-142
- EICHSTÄDT, S. et al. 2013. Comparison of the Richardson-Lucy method and a classical approach for spectrometer bandpass correction. *Metrologia*, 50,107-118
- POSSOLO, A. et al. 2010. Copulas for uncertainty analysis. Metrologia, 47, 262-271