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EVALUATION OF HUE SHIFT FORMULAE IN CIELAB AND CAM02

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Abstract

Calculated hue shifts could vary widely for spectrally similar light sources, which suggests a limitation of the current hue shift formulae for non-white light sources. Calculated differences in the hue appearance of 24 colour samples under 279,936 iteratively-generated test SPDs versus two reference illuminants were compared using four hue shift formulae that are based on two widely used colour spaces, CIE 1976 L^* a^* b^* (CIELAB) and colour appearance model 2002 (CAM02). Hue shift equations did not correlate well, even when only nominally “white” light sources were considered. Results suggest that the underlying difference between the hue shift formulae might be the non-uniform scaling of redness-greenness (a) and yellowness-blueness (b) coordinates in CIELAB and CAM02-UCS. A chromatic adaptation transform correction improved the correlation between lightness in CIELAB and CAM02-UCS when the test light sources were nominally “white,” but caused irregularities for non-white light sources. Further research is needed for visual assessment of the hue shifts and to evaluate the hue shift formulae.

Keywords: Colorimetry, Colour, Hue, Colour Appearance Model, Chromatic Adaptation

1 Introduction

The colour appearance of objects can be quantified using three-dimensional colour spaces. The three dimensions of a colour space are typically lightness, colourfulness, and hue. Lightness can be described as the brightness of a surface, and colourfulness indicates the strength of the chromatic content (e.g. redness of a red object). The International Commission on Illumination (CIE) describes hue as the “attribute of a visual perception according to which an area appears to be similar to one of the colours: red, yellow, green, and blue” (CIE, 2011), which is categorically different from lightness and colourfulness for inexperienced observers. Humans are typically more sensitive to hue shifts than changes in saturation and lightness (Danilova and Mollon, 2016). Although hue can be calculated using colour appearance models (CAMs)—mathematical models quantifying the perceptual attributes of coloured stimuli— hue is not widely used to quantify object colours in architectural lighting practice and research.

Hue angle (Δh) and hue (ΔH) differences can be calculated in two of most widely used colour spaces: CIE 1976 $L^*a^*b^*$ (CIELAB) and CAM02 (CIE, 2018). In both colour spaces, hue angle is calculated h = arctan (b/a), which is between 0 and 360 degrees and corresponds to the hue circle. Since a is the redness-greenness and b is the yellowness-blueness component, h = 0 approximates the appearance of red, h = 90 yellow, h = 180 green, and h = 270 blue object surfaces. The hue difference (ΔH) in CIELAB and CAM02-UCS is calculated using hue angle (h) and chroma values (CIE, 2018).

A recent study showed that the calculated hue differences (ΔH) can vary greatly even when the total calculated colour difference is small (Durmus and Davis, 2018). Although in this study large calculated hue shifts in CAM02 uniform colour space (CAM02-UCS) under a reference white illuminant and optimised non-white light source were found (Durmus and Davis, 2018), other studies did not find similar large hue shifts in CIELAB under a reference white illuminant and optimised non-white light sources (Durmus et al., 2018; Abdalla et al., 2016). It is possible that the reference illuminant and the colour space differences cause abnormality in hue shift calculations for highly structured spectral power distributions (SPDs), especially when the light source chromaticity deviates from the Planckian locus. Here, hue shift calculations under a large combination of test SPDs are compared to investigate the underlying issue.

2 Methods

Differences in hue was calculated for 24 Macbeth ColorChecker samples under test and reference conditions using four formulae, two formulae in each colour space (CIELAB and CAM02). Two reference SPDs were considered: CIE standard illuminant D50 and a white phosphor-converted LED (pcLED). Test SPDs were created by iteratively mixing seven narrowband channels of the Source Four LED Profile x7 Colour System™ in six steps of DMX control signal (0%, 20%, 40%, 60%, 80%, and 100%) for each LED channel, a total of 279,936 combinations. For example, one test SPD consisted of 0% LED channel 1, 20% LED channel 2, 60% LED channel 3, 80% LED channel 4, 100% LED channel 5, 0% LED channel 6, and 40% LED channel 7. All test SPDs were compared to each reference SPD. The radiant power of the test SPDs were adjusted to equalise the luminance of the sample in the reference and test conditions, to prevent changes in hue and chroma caused by luminance differences (Hunt and Pointer, 2011).

Hue shifts were calculated using four formulae, all based on Séve's hue shift calculations (1991). The first formula,

$$\Delta H^*_{Seve} = 2(C_1^* C_2^*)^{1/2} \sin(\Delta h / 2) \tag{1}$$

where

$C = (a^{*2}+b^{*2})^{1/2}$ is the chroma;

h is the hue angle;

is recommended by the CIE to calculate hue shifts (CIE, 2018). The second formula was an alternative method offered by Stokes and Brill (1992),

$$\Delta H^*_{SB} = s[2(Q - a_1^* a_2^* - b_1^* b_2^*]^{1/2} \tag{2}$$

where

$Q = C_1^* C_2^* = [(a_1^{*2}+b_1^{*2})(a_2^{*2}+b_2^{*2})]^{1/2}$

$$\text{if} \begin{cases} a_1^* b_2^* > a_2^* b_1^* \rightarrow s = 1, \\ otherwise \rightarrow s = -1, \end{cases}$$

to calculate hue shifts more efficiently (to reduce computation time) in CIELAB. The third and fourth methods were Séve's hue shift formula applied to CAM02. The typical CAM02 parameter settings (L_A = 60 cd/m^2, S_R = 1, surround = average) for the "surface colour evaluation in a light booth" (Luo and Li, 2013) were used to enable visual evaluations of the computational results in the future. Although a hue shift formula was not proposed in the original CAM02, hue shift formulae can be adapted to other colour spaces (Luo et al., 2006) and CAM02

$$\Delta H^*_{CAM02,mod1} = 2\sqrt{C_1 C_2} \sin(\Delta h / 2) \tag{3}$$

and

$$\Delta H^*_{CAM02,mod2} = 2\sqrt{M_1' M_2'} \sin(\Delta h / 2) \tag{4}$$

where

C is the chroma;

M' is the colourfulness.

ΔH^*_{Seve} and $\Delta H^*_{CAM02,mod1}$ are based on chroma, C, and hue angle, h (calculated using a-b and a'_M -b'_M coordinates, respectively). While chroma, and therefore ΔH^*_{Seve}, in CIELAB is solely dependent on a^* and b^* coordinates, CAM02 makes a distinction between colourfulness (M'), chroma (C), and saturation (s), and provides three outputs for this dimension that are not interchangeable (Luo and Li, 2013). The CIE defines chroma as “colourfulness of an area judged as a proportion of the brightness of a similarly illuminated area that appears white or highly transmitting” (2017). Since chroma is approximately constant across luminance changes, and colourfulness is an absolute perceptual quantity (i.e. changing with luminance) (Fairchild, 2013), it is possible that chroma (C^*) in CIELAB corresponds to colourfulness instead of chroma. However, $\Delta H^*_{CAM02,mod1}$ uses chroma instead of colourfulness. Unlike ΔH^*_{Seve}, in which chroma is not brightness dependent, in CAM02, the calculation of chroma includes lightness and the achromatic response, as well as a' and b' coordinates. It is likely that $\Delta H^*_{CAM02,mod1}$ is affected by the chroma-colourfulness distinction in CAM02. To address this, $\Delta H^*_{CAM02,mod1}$ was modified by replacing chroma with colourfulness in CAM02 ($\Delta H^*_{CAM02,mod2}$).

3 Results and discussion

3.1 Reference light source

The minimum, maximum, average and standard deviation of the hue shifts for each colour sample were calculated. The minimum, maximum and average hue shifts for each sample were then averaged over all samples, as shown in Table 1. The average hue shift over all colour samples was $\Delta H^*_{Seve,avg}$ = -0.3 ($\Delta H^*_{Seve,sd}$ = 51.8) when the reference was D50 and was $\Delta H^*_{Seve,avg}$ = -0.1 ($\Delta H^*_{Seve,sd}$ = 51.4) when the reference was the pcLED. The average hue shift was $\Delta H^*_{CAM02,mod1,avg}$ = -2.0 ($\Delta H^*_{CAM02,mod1,sd}$ = 43.8) when the reference was D50 and was $\Delta H^*_{CAM02,mod1,avg}$ = -2.5 ($\Delta H^*_{CAM02,mod1,sd}$ = 43.0) when the reference was the pcLED.

Table 1 –The minimum (min.$_t$) and maximum (max.$_t$) hue shift over all colour samples of all SPDs, average (avg.$_{avg}$), average of the minimum (avg.$_{min}$), and average of the maximum (avg.$_{max}$) hue shifts over all colour samples for the four hue shift formulae

	Reference D50					Reference pcLED				
	Min.$_t$	Max.$_t$	Avg.$_{avg}$	Avg.$_{min}$	Avg.$_{max}$	Min.$_t$	Max.$_t$	Avg.$_{avg}$	Avg.$_{min}$	Avg.$_{max}$
ΔH^*_{Seve}	-285	283	-0.3	-157	162	-372	332	0.1	-170	165
ΔH^*_{SB}	-280	257	-0.5	-123	111	-303	261	-1.6	-127	113
$\Delta H^*_{CAM02,mod1}$	-321	317	-2.0	-165	173	-351	414	-2.5	-170	161
$\Delta H^*_{CAM02,mod2}$	-116	114	-1.2	-71	72	-123	133	-1.5	-73	70

Calculated hue shifts varied slightly under two reference light sources. While the order of the range of hue shift difference was similar (i.e. $\Delta H^*_{CAM02,mod1}$ range was the greatest, $\Delta H^*_{CAM02,mod2}$ range was the smallest), minimum hue shift was ΔH^*_{Seve} when the reference was pcLED. However, the difference in minimum, maximum, and average values were not as great as the difference between hue shift formulae.

When calculate hue shifts were investigated under reference D50 and pcLED, no correlation was found between any of the hue shift equations for any sample. $\Delta H^*_{CAM02,mod1}$ varied more (from -321 to 317 when the reference was D50, and from -351 to 414 when the reference was pcLED) than ΔH^*_{Seve} (from -285 to 283 when the reference was D50, and from -372 to 332 when the reference was pcLED). The hue shift formula $\Delta H^*_{CAM02,mod2}$ showed the least variation (from -116 to 114 when the reference was D50, and from -123 to 133 when the reference was pcLED). Since highly chromatic lighting may cause large hue shift differences for low chroma surfaces (e.g. grey samples) (CIE, 2018), hue shifts for chromatic samples were compared to achromatic samples by grouping them according to their spectral reflectance shapes. A similar grouping was used in previous studies (Durmus and Davis, 2015a, 2015b, 2018), where *peak* were samples 3, 6, 11, 14, and 18, *plateau* were samples 1, 2, 7, 9, 12, 15, and 16, *peak+incline* were samples 4, 5, 8, 10, 13, 17, and *plain* were samples 19, 20, 21, 22, 23, and 24. Hue shifts for chromatic samples (peak, plateau, and peak+incline type reflectance) showed greater variation compared to achromatic samples (plain type reflectance), as shown in Table 2. These results are similar to previous findings (Durmus and Davis, 2018), which suggests that the hue shift anomalies are not due to axis-crossing (a-b axes) in low chroma object appearance.

Table 2 –The average (avg.$_{avg}$), average of the minimum (avg.$_{min}$), and average of the maximum (avg.$_{max}$) hue shifts for peak (samples 3, 6, 11, 14, and 18), plateau (samples 1, 2, 7, 9, 12, 15, and 16), peak+incline (samples 4, 5, 8, 10, 13, 17) and plain (samples 19, 20, 21, 22, 23, and 24) type reflectances for the four hue shift formulae

		Reference D50				Reference pcLED			
		ΔH^*_{Seve}	ΔH^*_{SB}	$\Delta H^*_{CAM02,mod1}$	$\Delta H^*_{CAM02,mod2}$	ΔH^*_{Seve}	ΔH^*_{SB}	$\Delta H^*_{CAM02,mod1}$	$\Delta H^*_{CAM02,mod2}$
Peak	Avg$_{.avg}$	0	-13	1	0	0	-8	-2	-1
	Avg$_{.min}$	-209	-151	-246	-98	-216	-151	-239	-98
	Avg$_{.max}$	192	201	245	95	205	196	233	93
Plateau	Avg$_{.avg}$	-1	13	-4	-2	0	10	-10	-6
	Avg$_{.min}$	-197	-194	-208	-91	-203	-209	-218	-93
	Avg$_{.max}$	217	132	240	98	192	138	213	91
Peak + incline	Avg$_{.avg}$	0	-5	-3	-2	0	-10	3	1
	Avg$_{.min}$	-205	-126	-203	-88	-244	-127	-215	-92
	Avg$_{.max}$	211	111	195	86	245	115	192	87
Plain	Avg$_{.avg}$	0	-1	0	0	0	-2	0	0
	Avg$_{.min}$	-20	-16	-10	-8	-20	-13	-12	-8
	Avg$_{.max}$	22	14	11	8	19	12	11	8

The explanation for the wider range of values from $\Delta H^*_{CAM02,mod1}$ could be the contribution of the lightness and achromatic response in the CAM02 hue calculations, since the reference and test a'_M and b'_M coordinates of CAM02-UCS varies less ($-118 < a'_M < 111$, $\Delta a'_{M,avg} = 0.2$, $-39 < b'_M < 50$, $\Delta b'_{M,avg} = 0.6$) compared to the a^* and b^* coordinates of CIELAB ($-218 < a^* < 532$, $\Delta a^*_{avg} = 13.3$, $-547 < b^* < 164$, $\Delta b^*_{avg} = -15.6$). On the other hand, the difference in these coordinates could also be attributed to the uniformity and scale differences between CIELAB and CAM02-UCS, especially in the positive a^* and negative b^* directions (Luo et al., 2006). For example, the CIELAB a^*-b^* coordinates ($-50 < a^* < 60$, $-50 < b^* < 60$) for the OSA data set (MacAdam, 1974) ranges slightly more compared to CAM02-UCS a'_M-b'_M ($-30 < a'_M < 40$, $-35 < b'_M < 45$) for the same data set (Luo et al.,2006). To test the impact of colour space uniformity and scale difference, calculated hue shifts for all SPDs were compared to those for only nominally “white” test light sources.

3.2 All test SPDs and nominally “white” test SPDs

When test SPDs were limited to nominally “white” light sources (close to the Planckian locus) according to ANSI specifications (ANSI, 2017), hue shifts calculated with CAM02-UCS were smaller than hue shifts calculated with CIELAB, as shown in Table 3. The range of lightness and *a*-*b* coordinates were similar under CIELAB and CAM02-UCS, as shown in Table 4. Calculated hue shifts for nominally “white” SPDs showed a smaller range compared to previous findings. However, when the test light sources were limited to be nominally “white,” the correlation between hue shift formulae did not improve.

Table 3 –The minimum ($min._t$) and maximum ($max._t$) hue shift over all colour samples of nominally “white” light sources, average ($avg._{avg}$), average of the minimum ($avg._{min}$), and average of the maximum ($avg._{max}$) hue shifts over all colour samples for the four hue shift formulae

	Reference D50					Reference pcLED				
	$Min._t$	$Max._t$	$Avg._{avg}$	$Avg._{min}$	$Avg._{max}$	$Min._t$	$Max._t$	$Avg._{avg}$	$Avg._{min}$	$Avg._{max}$
ΔH^*_{Seve}	-204	194	-1.5	-92	92	-207	195	-1.4	-93	91
ΔH^*_{SB}	-66	72	-0.1	-29	29	-60	71	-1.4	-30	28
$\Delta H^*_{CAM02,mod1}$	-137	139	1.02	-74	75	-148	140	1.3	-75	74
$\Delta H^*_{CAM02,mod2}$	-77	78	6.2	-45	46	-78	78	0.8	-45	45

When hue shifts were calculated for all SPDs, average lightness values were similar (J'_{avg} = 58, L^*_{avg} = 55). However, lightness in CAM02-UCS (J') ranged from 9 to 180, while lightness in CIELAB (L^*), which is mathematically limited from 0 to 100, ranged from 21 to 96. When hue shifts were calculated only for nominally “white” test light sources, lightness range in CAM02-UCS (J'_{avg} = 58, $24 < J' < 96$) was similar to lightness range in CIELAB (L^*_{avg} = 55, $21 < L^* < 95$).

Table 4 –The average, minimum and maximum CIELAB and CAM02-UCS coordinates under all test SPDs and nominally “white” test light sources

		CIELAB			CAM02-UCS		
		L^*	a^*	b^*	J'	a'_M	b'_M
All test SPDs	Avg.	55	-7	24	58	2	2
	Max.	96	532	164	180	111	50
	Min.	21	-218	-547	9	-118	-39
“White” test light sources	Avg.	55	6	8	58	2	2
	Max.	95	53	82	96	51	40
	Min.	21	-38	-55	24	-40	-34

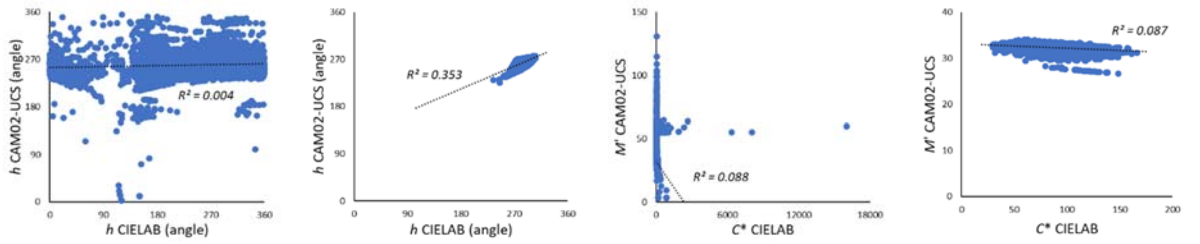
A previous study suggested that there are large mean hue angle (h) differences between CIELAB and CAM02 (especially for $190 < h < 290$) and concluded that CIELAB has good lightness uniformity with poor hue uniformity, and CAM02 performs well in colour difference but has poor lightness and hue uniformity (Jin et al., 2009). Results from computations presented here supported this idea where variation in maximum and minimum lightness in CIELAB was very small (L^*_{all}/L^*_{white} = ~1) between all SPDs and “white” light sources, but variations in maximum and minimum a^*-b^* coordinates were greater (between 2 and 10 fold) than variations in maximum and minimum a'_M-b'_M coordinates in CAM02-UCS (between 1 and 3 fold). Maximum and minimum lightness variations in CAM02-UCS were moderate ($0.4 < J'_{all}/J'_{white} < 1.87$), but poorer compared to CIELAB.

3.3 Chromatic adaptation transform correction

Hue angle anomalies at low tristimulus ratios have been previously reported for translucent objects (McLaren, 1980; Liu et al., 1995) and the use of chromatic adaptation transforms (CATs) are recommended for more accurate hue shift calculations (Li and Melgosa, 2012). A chromatic adaptation transform CMCCAT2000 (Li et al., 2002) was applied to CIELAB calculations to address this problem. CMCCAT2000 was found to outperform other CATs (Luo et al., 2003), and it has been previously used to minimise CIELAB non-uniformities (Davis and Ohno, 2010).

When test SPDs were limited to nominal “white,” hue angle (h) correlation between CIELAB and CAM02-UCS was improved. Figure 1 shows the hue angle and colourfulness differences between CIELAB and CAM02-UCS for a blue sample (sample 13) when the reference is CIE illuminant D50. Hue angles (h) were more varied in CIELAB compared to CAM02-UCS, especially when all SPDs were considered. Although hue angle in CAM02-UCS covered most of the hue circle (360 degrees) when all SPDs were considered, they were mostly concentrated on the dominant hue angle (e.g. h = 270 which approximates blue, as shown in Fig. 1a). Limiting the test light sources to close to the Planckian locus region decreased the calculated hue angle ranges both in CAM02-UCS and CIELAB.

Figure 1 – Hue angle ((a) and (b)) and colourfulness ((c) and (d)) values in CIELAB and CAM02-UCS calculated for all test SPDs ((a) and (c)) and nominally "white" light sources ((b) and (d)) respectively for sample 13

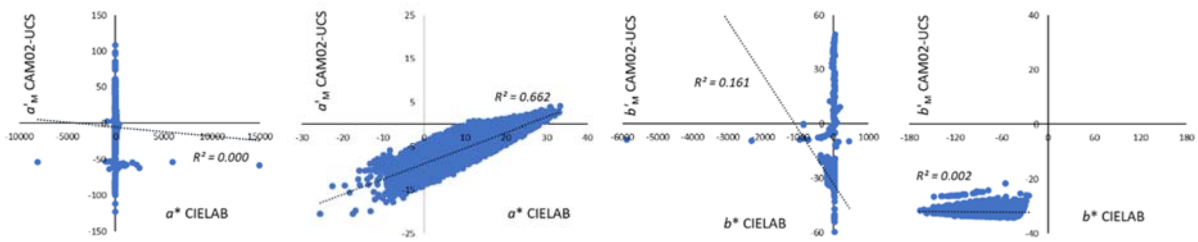


(a) (b) (c) (d)

Similar to the hue angle, colourfulness in CIELAB showed larger variations compared to colourfulness in CAM02-UCS. Unlike the hue angle, limiting test SPDs to nominal "white" did not improve the correlation between colourfulness in CIELAB and CAM02-UCS.

In addition to the hue angle (*h*) and colourfulness differences, lightness, redness-greenness (*a*), yellowness-blueness (*b*) were compared to identify the underlying difference between hue shifts in CIELAB and CAM02-UCS based formulae. An example in Fig. 2 shows the correlation between redness-greenness (*a*), yellowness-blueness (*b*) in CIELAB and CAM02-UCS for a blue sample (sample 13) when the reference is CIE illuminant D50. Both redness-greenness (*a*) and yellowness-blueness (*b*) coordinates ranged more in CIELAB compared to CAM02-UCS when all test SPDs were considered. When test SPDs were limited to nominal "white," *a*-*b* coordinate ranges were similar. When test SPDs were limited to nominal "white," correlation between redness-greenness (*a*) improved (e.g. R^2 = 0.662 for sample 13) more than yellowness-blueness (*b*) correlation (e.g. R^2 = 0.002 for sample 13). The variation differences in coordinates (e.g. $a^*_{max}/a'_{M,max}$ ~= 140, $a^*_{min}/a'_{M,min}$ ~= 65, $b^*_{max}/b'_{M,max}$ ~= 12, $b^*_{min}/b'_{M,min}$ ~= 100 for sample 13) suggests that if there is a scale difference between two colour spaces, it is not linear.

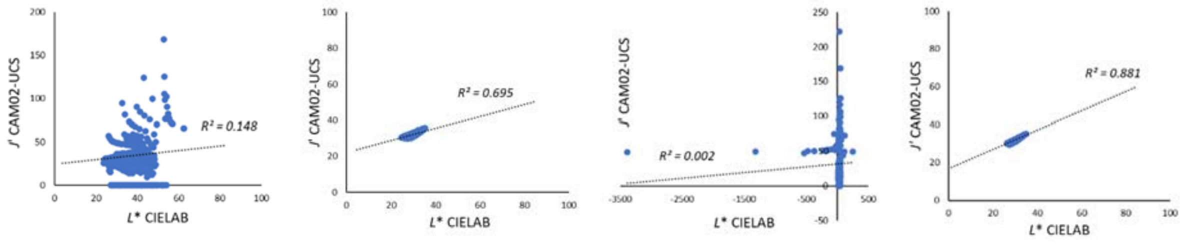
Figure 2 – Redness-greenness coordinates ((a) and (b)) and blueness-yellowness coordinates ((c) and (d)) in CIELAB and CAM02-UCS calculated for all test SPDs ((a) and (c)) and nominally "white" light sources ((b) and (d)) respectively for sample 13



(a) (b) (c) (d)

The correlation between CIELAB and CAM02-UCS lightness values were poor when all test SPDs were considered. Lightness in CIELAB (L^*) were not limited between 0 and 100 due to the CMCCAT2000 correction. An example is given Fig 3. which shows the difference in lightness values between previous results and CAT corrected calculations across colour spaces for a blue sample (sample 13). In previous calculations, CIELAB and CAM02-UCS lightness correlation was low (e.g. R^2 = 0.148 for sample 13) when all test SPDs were considered. Limiting test SPDs to nominally "white" increased the coefficient of determination (e.g. R^2 = 0.695 for sample 13). When CAT correction was applied, the coefficient of determination increased even more (e.g. R^2 = 0.881 for sample 13). The results suggest that CAT corrections increase the correlation in lightness between CIELAB and CAM02-UCS for only nominally "white" light sources.

Figure 3 – Lightness in CIELAB (*L) and CAM02-UCS (*J'*) calculated for all SPDs ((a) and (c)) and nominally "white" light sources ((b) and (d)) respectively, for sample 13, with ((c) and (d)) and without ((a) and (b)) chromatic adaptation transform correction**



(a) (b) (c) (d)

4 Conclusion

Calculated hue shifts for chromatic samples may vary significantly under very similar test SPDs (e.g. $\Delta H^{*}_{CAM02,mod1}$ = 29 for two spectra that has a "good" spectral match according to goodness-of-fit coefficient GFC= 0.9994 (Hernandez-Andres et al., 2001)), which suggests a shortcoming of the hue shift formulae. To address this issue, hue shift calculations were performed in CIELAB and CAM02 with a large number of test SPDs and standard illuminant D50 and white pcLED references. Colorimetric calculations for all test SPDs and nominally "white" test light sources were compared to investigate the impact of colour space uniformity and scale on calculated hue shifts.

The results show that hue shift equations do not correlate well, even when light sources were limited to near-Planckian locus region (nominal "white") and a chromatic adaptation transform was applied to CIELAB calculations. Results from the computations support the idea that colour space uniformity and scale might impact calculated hue shifts. The possible scale differences between redness-greenness (*a*) and yellowness-blueness (*b*) coordinates along the colour spaces were not linear. Lightness in CIELAB and CAM02 correlated well for nominally "white" light sources, especially when a CAT correction was applied to CIELAB. However, CAT correction caused irregularities (i.e. *L** ranging beyond 0 and 100) when test SPDs were not close to the Planckian locus. Future work will investigate the visual assessment of hue shifts and test the performance of hue shift formulae.

References

ABDALLA, D., DUIS, A., DURMUS, D. and DAVIS, W. 2016. Customisation of light source spectrum to minimise light absorbed by artwork. *Proceedings of the CIE Lighting Quality and Energy Efficiency*, pp. 22-31. CIE x042:2016

ANSI. 2017. *C78.377-2017 American National Standard for Electric Lamps—Specifications for the Chromaticity of Solid-State Lighting (SSL) Products*, (National Electrical Manufacturers Association).

CIE. 2018. CIE 15:2018. *Colorimetry*. Vienna: CIE. DOI: 10.25039/TR.015.2018

CIE. 2011. CIE S 017/E:2011. *International Lighting Vocabulary.* Vienna: CIE.

DANILOVA, M.V. and MOLLON, J.D. 2016. Superior discrimination for hue than for saturation and an explanation in terms of correlated neural noise. *Proceedings of the Royal Society B: Biological Sciences*, *283*(1831), pp. 20160164. DOI: 10.1098/rspb.2016.0164

DAVIS, W. and OHNO, Y. 2010. Color quality scale. *Optical engineering*, *49*(3), 033602. DOI: 10.1117/1.3360335

DURMUS, D., ABDALLA, D., DUIS, A. and DAVIS, W. 2018. Spectral optimization to minimize light absorbed by artwork. *Leukos*, 1-10. DOI: 10.1080/15502724.2018.1533852

DURMUS, D. and DAVIS, W. 2015a. Absorption-Minimizing Spectral Power Distributions. *OSA Technical Digest (online) (Optical Society of America, 2015),* paper JTu5A.2. DOI: 10.1364/PV.2015.JTu5A.2

DURMUS, D. and DAVIS, W. 2015b. Optimising light source spectrum for object reflectance. *Optics Express*, *23*(11), pp.A456-464.DOI: 10.1364/OE.23.00A456

DURMUS, D. and DAVIS, W. 2018. Appearance of Achromatic Colors Under Optimized Light Source Spectrum. *IEEE Photonics Journal*, *10*(6), pp.1-11.DOI: 10.1109/JPHOT.2018.2881698

FAIRCHILD, M. D. 2013. *Color appearance models, 3th ed*. Hoboken, NJ: John Wiley & Sons.

HERNANDEZ-ANDRES, J., ROMERO, L. and LEE, R. L. 2001. Colorimetric and spectroradiometric characteristics of narrow-field-of-view clear skylight in Granada, Spain. *JOSA A*, *18*(2), pp. 412-420. DOI: 10.1364/JOSAA.18.000412

HUNT, R. W. G. and POINTER, M. 2011. *Measuring Colour, 4th ed*. Hoboken, NJ: John Wiley & Sons.

JIN, H., ZHAO, X. and LIU, H. 2009. Testing of the uniformity of color appearance space. In *WRI World Congress on Computer Science and Information Engineering* (vol. 6, pp. 307-311). IEEE, Los Angeles, CA. DOI: 10.1109/CSIE.2009.539

LI, C., LUO, M. R., RIGG, B. and HUNT, R. W. 2002. CMC 2000 chromatic adaptation transform: CMCCAT2000. *Color Research & Application*, *27*(1), 49-58. DOI: 10.1002/col.10005

LI, C. and MELGOSA, M. 2013. A note about the abnormal hue angle change in CIELAB space. *Color Research & Application*, *38*(5), 322-327. DOI: 10.1002/col.21733

LIU, Y., SHINGLEY, J., FRITSCH, E. and HEMPHILL, S. 1995. Abnormal Hue-Angle change of the gemstone tanzanite between CIE illuminants d65 and a in CIELAB color space. *Color Research & Application*, *20*(4), 245-250.DOI: 10.1002/col.5080200407

LUO, M.R., CUI, G. and LI, C. 2006. Uniform colour spaces based on CIECAM02 colour appearance model. *Color Research & Application*, *31*(4), pp.320-330. DOI: 10.1002/col.20227

LUO, M.R. and LI, C. 2013. CIECAM02 and its recent developments. In *Advanced Color Image Processing and Analysis* (pp. 19-58). New York, NY: Springer. DOI: 10.1007/978-1-4419-6190-7_2

LUO, M. R., RIGG, B. and SMITH, K. J. 2003. CMC 2002 colour inconstancy index; CMCCON02. *Coloration Technology*, *119*(5), 280-285. DOI: 10.1111/j.1478-4408.2003.tb00184.x

MACADAM, D. L. 1974. Uniform color scales. *JOSA*, *64*(12), pp. 1691-1702. DOI: 10.1364/JOSA.64.001691

MCLAREN, K. 1980. CIELAB hue-angle anomalies at low tristimulus ratios. *Color Research & Application*, *5*(3), 139-143. DOI: 10.1002/col.5080050306

SÉVE, R. 1991. New formula for the computation of CIE 1976 hue difference. *Color Research & Application*, *16*(3), pp.217-218. DOI: 10.1002/col.5080160311

STOKES, M. and BRILL, M.H. 1992. Efficient Computation of ΔH* ab. *Color Research & Application*, *17*(6), pp.410-411. DOI: 10.1002/col.5080170608