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Commission Internationale de l'Eclairage
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# TECHNICAL NOTE

## Chromaticity Difference Specification for Light Sources

**CIE TN 001:2014**

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## 1 Introduction

Chromaticity difference of light sources needs to be specified in many cases, for example for angular colour uniformity, shift of colour during lifetime (colour maintenance) and individual colour variation of products. The chromaticity difference of light stimuli, in general, is expressed by the distance in the CIE 1976 uniform chromaticity scale (UCS) diagram (CIE, 2004; ISO/CIE, 2009), known as CIE ($u'$,$v'$) chromaticity diagram or simply CIE ($u'$,$v'$) diagram, which represents the uniform chromaticity space for light stimuli officially recommended by the CIE. The term "chromaticity difference" should not be confused with colour difference of object colours, which is measured as the Euclidean distance in a three-dimensional object colour space such as CIE $L^*a^*b^*$ (also called CIELAB) (CIE, 2004; ISO/CIE, 2008) or with a colour difference formula such as CIEDE2000 (ISO/CIE, 2014), which requires a white reference.

## 2 Background

The chromaticity difference of light sources for illumination has often been expressed by the size of MacAdam ellipses (MacAdam, 1942). For example, Figure 1 shows the chromaticity tolerance specification for linear fluorescent lamps by the IEC (1997). These ellipses are so-called five-step (or 5-SDCM[1]) MacAdam ellipses, which are five times the size of the original MacAdam ellipses interpolated for the given centre points. Similar specifications are used in USA (ANSI, 2001), where four-step MacAdam ellipses are specified with a slight difference in the centre point for 2 700 K. The size of the original MacAdam ellipses is defined as 1 SDCM. However, the sizes and shapes of MacAdam ellipses plotted in the CIE 1931 ($x$,$y$) or 1976 ($u'$,$v'$) chromaticity diagram are all different (see Figure 2) and thus 1 SDCM is not constant. For a perfect uniform colour space, all ellipses should be constant sized circles. It can be seen that all ellipses close to the Planckian locus are closer to circles in the ($u'$,$v'$) diagram than those in the ($x$,$y$) diagram. Note that MacAdam's experiments were conducted with one condition of surrounding field approximating CIE Illuminant C (~6 800 K standard daylight, now obsolete). The ellipses may be different for adaptation to surrounding fields with other correlated colour temperatures.

Another expression for the deviation of chromaticity, occasionally used in the industry, is JND (Just Noticeable Difference), but its concept is not clearly defined or universally agreed in any standards or available scientific literature. In MacAdam's article, standard deviations of colour matching are plotted as the distances from the centre point of each ellipse. The relationship between the standard deviations of colour matching and JND is not clearly determined in that article, but MacAdam implied that JND is about three times the standard deviation. This factor, however, depends on the definition of JND. If it is defined as the chromaticity difference detected at 50 % probability, then assuming MacAdam's results follow Gaussian distribution (where 50 % probability occurs at 1,18 times the standard deviation), JND at 50 % probability would be 1,18 times the distance from the centre point of MacAdam ellipses. In any case, the definition of "just noticeable difference" is not standardized and is often confused.

While the MacAdam ellipses served for many years for fluorescent lamps with only six nominal correlated colour temperatures (CCTs), this method does not serve well for lamps using new lighting technologies, which are more versatile. For example, the ANSI chromaticity specifications for solid-state lighting products (ANSI_NEMA_ANSLG, 2011) define eight nominal CCTs and also flexible CCTs at 100 K intervals. If the chromaticity specification were to be given by MacAdam ellipses, their exact shape and size for the given centre points are unknown and would need to be interpolated from the original 25 ellipses measured by MacAdam (Figure 2). Such interpolation formulae are not defined in any international standard, though interpolation for other chromaticities using three parameters was discussed in his later article (MacAdam, 1943). This is a problem for LED light sources that do not follow the traditional nominal CCTs of fluorescent lamps. Even if interpolation of the ellipses can be specified, it would be complicated and inconvenient for practical use.

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[1] SDCM is the acronym for "**S**tandard **D**eviation of **C**olour **M**atching"

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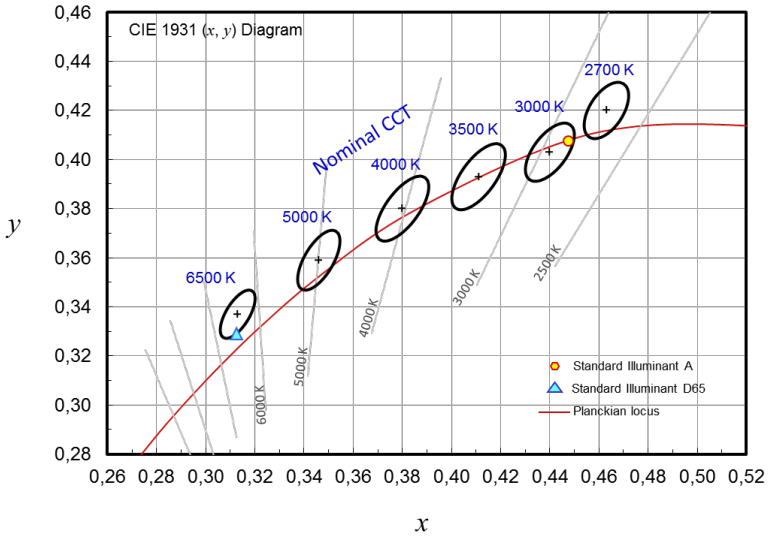


**Figure 1 — Specification for the chromaticity tolerances of linear fluorescent lamps as given in IEC 60081 using five-step MacAdam ellipses (IEC, 1997)**

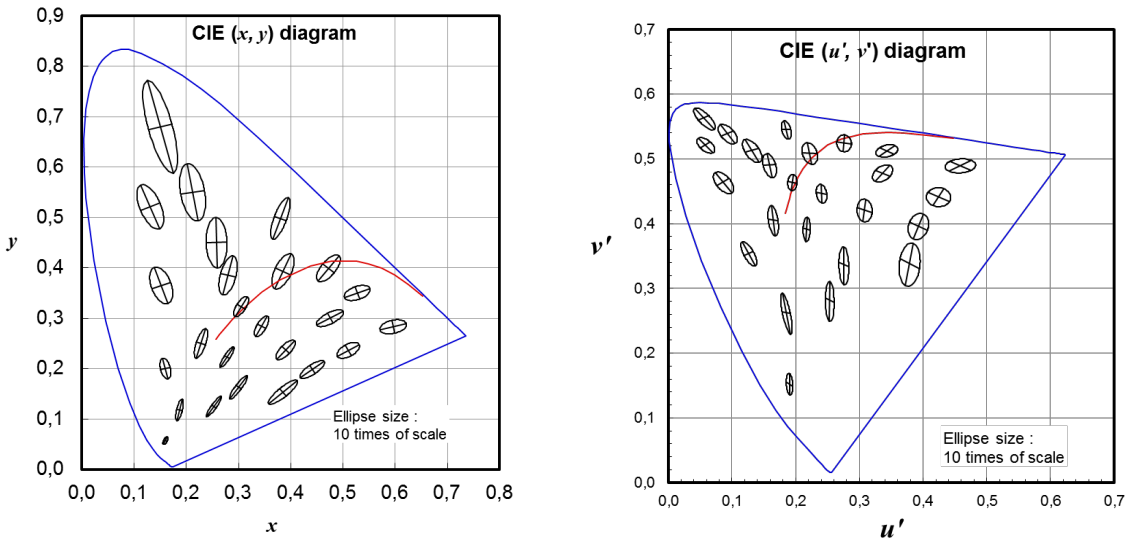


**Figure 2 — MacAdam ellipses (10 times of the scale) plotted in CIE 1931 ($x$,$y$) chromaticity diagram (left) and CIE 1976 ($u'$,$v'$) chromaticity diagram (right)**


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## 3 Alternative to MacAdam Ellipses

### 3.1 *u'v'* Circle

As is widely known, the CIE $(u',v')$ chromaticity diagram represents the most uniform colour space for light sources recommended by the CIE. Figure 3 shows the five-step MacAdam ellipses (solid black line) plotted in the $(u',v')$ diagram, on top of which circles with radius of 0,0055 (dashed red line) are plotted. As shown, these ellipses and the circles are closely overlapping, which means that circles can be used to replace MacAdam ellipses in this white region near the Planckian locus in the $(u',v')$ diagram. Five-step MacAdam ellipses are approximately equivalent to the circles with radius 0,0055 in $(u',v')$. The $u'v'$ *circle* is specified with a centre point $\left(u'_c,v'_c\right)$ and radius $r$ on the $(u',v')$ diagram, and expressed by,

$$\left(u'-u'_c\right)^2+\left(v'-v'_c\right)^2=r^2 \tag{1}$$

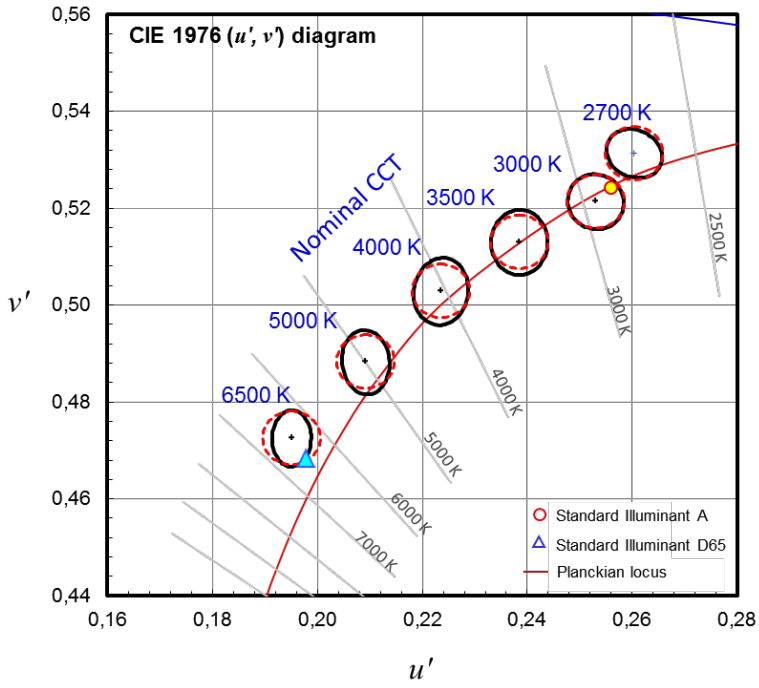


**Figure 3 — Five-step MacAdam ellipses in IEC 60081 (IEC, 1997) and circles (radius 0,0055) in the CIE 1976 (*u'*,*v'*) chromaticity diagram**

### 3.2 *n*-step *u'v'* circle

For consistency with MacAdam ellipses, the term "$n$-step $u'v'$ circle" is defined as a circle in the $(u',v')$ diagram with a radius of $n$ times 0,0011. For a centre point $\left(u'_c,v'_c\right)$, the $n$-step $u'v'$ circle is expressed by the equation:

$$\left(u'-u'_c\right)^2+\left(v'-v'_c\right)^2=\left(0{,}0011\cdot n\right)^2 \tag{2}$$

and corresponds to a $n$-step MacAdam ellipse. A just noticeable chromaticity difference at 50 % probability is considered to be (0,0011 × 1,18 =) 0,0013 in $(u',v')$ coordinates. Note that this applies only to the white region of the $(u',v')$ diagram.

The centre points of the six MacAdam ellipses of IEC 60081 in $(u',v')$ coordinates, thus the centre points of the corresponding 5-step $u'v'$ circles, are given in Table 1.

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**Table 1 — ($u'$,$v'$) coordinates of the centre points of the $u'v'$ circles corresponding to the six MacAdam ellipses in IEC 60081 (IEC, 1997)**

| Nominal CCT | $u'$ | $v'$ |
|---|---|---|
| 2 700 K | 0,2603 | 0,5313 |
| 3 000 K | 0,2530 | 0,5214 |
| 3 500 K | 0,2385 | 0,5131 |
| 4 000 K | 0,2235 | 0,5029 |
| 5 000 K | 0,2092 | 0,4884 |
| 6 500 K | 0,1951 | 0,4726 |

There are small differences between the $u'v'$ circles and MacAdam ellipses observed in Figure 3, but they are considered insignificant in practical applications, especially when considering the experimental uncertainty of MacAdam ellipses derived 70 years ago with only one experimental subject and with only one illuminant for the surrounding field.

## 4 Chromaticity difference $\Delta_{u',v'}$ [1]

Chromaticity differences of light sources (any colour) are generally expressed by the distance in ($u'$,$v'$) coordinates, which is denoted as $\Delta_{u',v'}$ and is calculated as the distance between two points $(u'_1,v'_1)$ and $(u'_2,v'_2)$ on the ($u'$,$v'$) diagram by

$$\Delta_{u',v'} = \sqrt{(u'_2 - u'_1)^2 + (v'_2 - v'_1)^2} \ . \qquad (2)$$

The chromaticity difference in $(x,y)$ coordinates should not be used for this purpose because it does not represent the visual perception of chromaticity differences.

## 5 Recommendation

For specifying chromaticity tolerances for light sources for general lighting it is recommended that $u'v'$ circles be used rather than MacAdam ellipses. For solid state light sources, if not using $u'v'$ circles, quadrangles may also be used. Also, to specify chromaticity uniformity (changes over angle) and long-term chromaticity maintenance (changes over time), it is recommended that a $u'v'$ circle or $\Delta_{u',v'}$ be used. The use of Standard Deviation of Colour Matching (SDCM) and Just Noticeable Difference (JND) as units of chromaticity difference is discouraged. Note that $u'v'$ circles are recommended only for light sources with chromaticity close to the Planckian locus, while chromaticity differences $\Delta_{u',v'}$ may be used for light sources with any other chromaticities.

## References

ANSI., 2001. C78.376 *Specifications for the Chromaticity of Fluorescent Lamps*.

ANSI_NEMA_ANSLG. 2011. C78.377-2008 *Specifications for the Chromaticity of Solid State Lighting Products*.

CIE. 2004. CIE 15:2004 *Colorimetry, 3rd edition*.

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[1] Note that the symbol for chromaticity difference given here follows the rules defined for the notation of quantity symbols and is therefore the symbol now recommended by CIE. It differs from the symbol used in ISO 11664-5:2009(E)/CIE S 014-5/E:2009 (ISO/CIE, 2009); the latter will be updated to use this new symbol when it is next reviewed.

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